Why Horizontal Inequalities Matter: Some implications for measurement

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Abstract

This paper considers why group inequalities (horizontal inequalities - HI) matter, as well as individual inequality (vertical inequality - VI). It argues that HIs matter both from the perspective of the wellbeing of individuals within groups who mind about how their group is faring relative to others, and instrumentally, through the impact of group inequalities in reducing growth potential and provoking violence. The paper reviews a set of measures for HI. We explore the correlation among selected measures of vertical and horizontal inequality in Indonesia using censal survey data and show that there is very high correlation among the VI measures and high correlation among the HI measures, while the correlation between HI and VI measures is less clear. Using data over time for South Africa and the United States we illustrate differences between alternative HI measures. The most appropriate measure depends on the purpose for which it is intended. For empirical research on the consequences of group inequalities, the more descriptive measures are preferable. We conclude that group Gini's and the group coefficient of variation weighted by the population size of the group are to be preferred from this perspective. However, in some contexts a simple ratio of group performance among the two groups of interest may be most informative.

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Why Horizontal Inequalities Matters: Some implications for measurement

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Most discussion and measurement of inequality concerns vertical inequality, or inequality among individuals, (VI), and is generally confined to a few economic variables, such as income, consumption, and sometimes assets. It is in this arena that the application of Lorenz curves and the Gini coefficient have been extensively and powerfully used. In so doing, group inequality (defined here as horizontal inequality - HI) tends to be ignored. We argue that it is important to measure such inequality; and that its measurement should extend beyond the variables commonly used to measure vertical inequality not only to other aspects of economic resources, but to social and political entitlements as well. We shall consider alternative ways of measuring horizontal inequalities and provide some empirical applications of different measures in the case of Indonesia, showing how far the different measures correlate with one another.

The next section of this paper will discuss why and when group inequalities matters; section 2 discusses definitional issues in more depth, and consider some conceptual and measurement problems; section 3 illustrates alternative measures in a few empirical cases; section 4 concludes.

1. What horizontal inequalities are and why they matter

Horizontal inequalities are inequalities between groups. People can be grouped in many ways, and most people are members of many groups. There is a large range of types of groups: national, racial, ethnic, religious, gender and age are some obvious important ways that people are categorised. In some case the categorisation emerges largely from self-identification, in others, classification comes from legal factors (such as citizenship), as a result of categorisation by others, or some combination. There are also many relatively transient types of group - such as social clubs, or producer networks, and so on. Some group affiliations are clearly more important than others.

Group affiliation matters both instrumentally and for well-being, particularly, when:

- group boundaries are relatively tight, so people cannot move easily (sometimes at all) from one group to another. An example is being of one gender; another is being a citizen of a particular country. If it is easy to change groups then the affiliation matters much less;
- being a group member leads to different treatment by others - e.g. via discrimination at many levels (in the gender case, this might start even within

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1 Income group or class is one important way in which people may be classified. However, since this type of classification is broadly captured by vertical inequality we shall not deal with it here, unless membership of different income groups is broadly the same as some other group classification (such as race).

2 ‘Always remember that you are a proud citizen of Prussia, entitled to equal rights. And never forget that you are a Jew. If you do there will always be others to remind you of your origins’ (advice given by a father to his son in the mid-19th century; Frister,2002: 58).This quotation underlines the fact that many people have multiple identities; and that group categorisation is partly a matter of self-identification, partly identification by others.
In the case of Africans in Apartheid South Africa, there was cradle to grave discrimination with political as well as economic dimensions. Privileges for particular groups also enhances the importance of group membership; and

- members of the group feel that being part of the group constitutes a significant aspect of their identity, and thereby group achievements contribute directly to members' well-being.

Group inequality (or HI) can be important both instrumentally, as a means of achieving other objectives, and in themselves (Loury, 1988). The direct impact on members' well-being is one most important aspect. People's well-being may be affected not only by their individual circumstances, but also by how well their group is doing. This is partly because membership of the group is part of a person's identity, and partly because relative impoverishment of the group increases perceptions of members that they are likely to be trapped permanently in a poor position, or, if they have managed to do better than many in the group, that they are likely to fall back into poverty. Hence it seems likely that the well-being of Moslems in Western Europe, Catholics in Northern Ireland, Hutus in Rwanda, Afro-Americans in the US, Africans in Apartheid South Africa, to take just a few of many examples, is (was) deeply affected by the relatively impoverishment of the group – which encompasses many dimensions – over and above the position of the individual themselves. Psychologists have shown, for example, that Afro-Americans suffer from many psychological ills due to the position of their group. This is summarised by the title of one article: 'Being Black and Feeling Blue' (Brown et al, 1999; Broman, 1997). Hence the relative position of the group should enter into a person's welfare function (Akerlof & Kranton, 2000). The weight to be given to this element is an empirical matter on which more research is needed.

There are several ways in which reducing HIs may be instrumental for the achievement of other objectives. One is by promoting efficiency: any system in which a group is discriminated against is likely to lead to less efficiency than in the absence of discrimination, since talented people in the group discriminated against will be held back, while too many resources, or too high a position, will go to less talented people in the favoured group. For example, Macours (2004) has argued that ethnic diversity in a context of weak property rights enforcement can result in market segmentation and less than optimal land allocation. In Guatemala, informal land contracts are more likely to take place within the same ethnic group. Conversely, most studies show that affirmative action for Afro-Americans in the US have had positive impact on efficiency (Badgett and Hartman, 1995). Disproportionately limited opportunities may arise as a result of formal discrimination in education or employment, for example. However, generations of relative impoverishment (possibly due to past discrimination, possibly to some random factors) mean that the present generation of a group may suffer relatively to others without current overt discrimination. This is because: (i) family background including nutrition and educational levels influence a child's chances in life; (ii) social networks operate disproportionately within a group and less between groups – indeed Blau (1977) regards having more in-group than out-group interactions as a defining characteristic of a group.

Consequently, a member of a poor group has less advantageous social networks (or good contacts). The social networking point is less important where membership of the various groups an individual belongs to are not coterminous. For example, where particular social groups are multiracial or multiethnic, on the assumption that every member of the latter type of group is treated equally – in practice, however,
theoretically multiracial social groups in fact often exercise segregation within the group.

A second type of instrumental reason for concern with HIs is that it may not be possible to improve the position of individuals without tackling the position of the group. For example, programmes to advance credit to poor producers, or to promote universal education, may not be achievable so long as group inequality remains. An example here is extending education to all girls which may be prevented not by a lack of schools or teachers, but by parental attitudes to girls' education (Hafeez, UNICEF & South Asian Association for Regional Cooperation, 1990; United States Information Service & United States Agency for International Development, 1992).

The third instrumental reason is that group inequality can be a source of violent conflict (Stewart, 2000). Group inequality provides powerful grievances which leaders can use to mobilise people to political protest, by calling on cultural markers (a common history or language or religion) and pointing to group exploitation. This type of mobilisation seems especially likely to occur where there is political as well as economic inequality, so that the leaders are excluded from political power. Examples where group inequalities have been a factor in provoking conflict include Cote d'Ivoire, Rwanda, Northern Ireland, Chiapas, the Sudan to mention just a few (see Gurr, 1993; Stewart, 2002; Gurr & Moore, 1997; Langer, 2005; Murshed and Gates, 2004). Sharp horizontal inequalities within countries (and between them) are an important source of grievance and potentially of instability, independently of the extent of vertical inequality. However, we might expect a non-linear relationship (inverse U-shaped) between the size of inequalities and conflict. Psychological research suggests that when the status of the privileged group is much higher than the status of the underprivileged group, members of the former show magnanimity towards the underdog (Hewstone et al., 2000: 585). According to systems theory, groups will be more likely to challenge each other when there is greater parity of resources. This in turn increases group beliefs in their chances of winning (Ellingsen, 2000).

HIs are a component of vertical inequality (VI); VI in any society can be decomposed into two elements: between group inequality and within group inequality (see B5, Appendix). Clearly, then vertical inequality is likely to be greater the higher HI. But it is possible to have considerable HI with rather little VI, where within group inequality is relatively low, or conversely. Where HI forms a big component of VI, reducing VI may be difficult without reducing HI. However, typically the between group component of total VI is small relatively to within group inequality, (e.g. Papatheodorou, 2000; Zhang and Kanbur, 2003; Pradhan et al., 2001). An important issue is the extent of the correlation between HI and VI. This will be investigated further in section III of this paper.

It is interesting to consider how far the reasons normally put forward for concern with VI, are the same as those we have noted for HIs. To analyse this, we need to consider why there should be concern for VI in a homogeneous society (i.e. without significant group differentiation).

Concern with VI arises for instrumental and welfare reasons as with HIs. Instrumentally, empirical research suggests that high VI seems to be negatively correlated with economic growth (e.g. Persson and Tabellini, 1994; Perotti,1993). Of course, this research includes the HI contribution to VI so does not say what happens in a homogeneous society. The fact that ethnically diverse societies have been found to have lower growth than homogeneous ones (Easterly and Levine, 2000; Montalvo and Reynal-Querol, 2005) suggests that group inequality is one component
of this growth lowering impact of inequality. The connection between inequality and growth has been attributed to the political impact of inequality, and to its human capital effects (among other explanations).\(^3\) Both these arise with HI also, but more powerfully because HIs tend to be durable – an individual finds it difficult/impossible to escape (Tilly, 1998). This is likely to be less true of VI in a homogeneous society because it is group characteristics that tend to trap people. Secondly, VI is of concern because it is difficult to achieve the poverty reduction objective without tackling it since at any given societal income level, more redistribution will raise the incomes of the poor (Birdsall and London, 1997). As noted above, where HIs are strong it may also be difficult to achieve poverty reduction without explicitly addressing group inequality. In this case, it is not only because of the arithmetic – i.e. that redistribution allows more poverty reduction for any given GNP – but also because it may be difficult to find effective mechanisms of redistribution without a direct attack on discrimination.

From the point of view of maximising utility, it is often suggested that Individuals’ relative position matters to them as well as their absolute position. Empirical research has investigated how far self-assessed happiness correlates with relative as well as absolute position with mixed results, but on the whole, in developing countries and Europe, relative impoverishment seems to be a source of unhappiness, although not in the US where inequality is argued to signal opportunity (Alesina, Di Tella, & MacCulloch, 2004; Ball & Chernova, 2004).

Whereas in homogenous societies, the impact on happiness would be just a question of the individual's position relative to others, in heterogeneous societies, this is also a factor, but in addition the inequality of the group may be another factor, as argued above. Some research has explored some group effects, although not the impact of group inequality directly. For example, Kingdon & Knight (2004) found that income of others in the local area has positive effect on happiness, while Graham & Pettinato (2002), for Peru, ‘find that respondents tend to be more critical of their economic situation when they compare themselves to others in their country than when they compare themselves to others in their community’ (cited in Graham & Felton, 2005: 9). Studies in stratified developing countries (Kingdon and Knight, 2004 for South Africa; Graham and Pettinato, 2002 for Peru) have found that the average income of a local reference group has a positive impact on people's happiness, while the impact of average incomes of more distant groups is negative, in addition to the effects of individuals’ own absolute and relative position. This broadly supports the view that the performance of a person's own identity group may affect their happiness positively while that of other identities groups may have a negative impact. Empirical research into the correlates of happiness has mostly focussed on individual not group inequality, but some studies in stratified countries have found that the incomes of local reference groups have a positive impact on individual happiness. However, more empirical research is needed on this issue.

In summary, then, some of the reasons for concern with individual inequality – both instrumental and well-being reasons – in fact stem partly from group inequality. There is reason to suppose that group inequality is a worse impediment to growth and makes the achievement of social objectives more difficult than individual inequality does in homogeneous societies. Group inequality is also more likely to be associated with conflict. While cross-country econometric investigations have given little support for the view that VI as such contributes to conflict (Collier and Hoefller, 2000; Collier, 2007; Declercq et al., 2008).

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Lichbach, 1989), there is more evidence that group inequality is associated with conflict (Gurr, 1993; Gurr et al., 1997; Østby, 2003; Mancini, 2005).

Inequality has many dimensions. Economists tend to measure inequality in terms of incomes or consumption, although Sen has advocated adopting the space of capabilities. (Sen, 1980). The appropriate space depends on what we are trying to assess. If it is well-being, income or consumption are very poor proxies since in general they do not include access to public goods, nor allow for distribution within the family, nor for the varying ways in which a given amount of income translates into capabilities (what people can be or do). Hence the much more extensive multidimensional space of capabilities is preferable.

However, in practice capabilities (as potentials) are not easily measured, so inequality of functionings (what people actually are or are doing) is generally the only available measure. Capabilities, or indeed functionings, are the consequence of all sorts of circumstances - i.e. of possession or access to a variety of assets, to employment opportunities, the availability and quality of publicly provided goods and the capacity and constraints faced by the individual. While inequality of the outcome (functionings) is of concern, so is inequality of some of the more significant inputs - partly because this helps to diagnose the source of outcome inequality, and partly because such inequality contributes directly to people's well-being. For example, inequality in land ownership contributes to people's ability to be nourished and so on, but it also contributes directly to their self-respect, status and well-being.

To the extent that we are concerned with the political impact of such inequalities, what is important is the elements that seem most significant to the people concerned, i.e. what they mind about. Evidence suggests that this differs across societies and groups: in Northern Ireland, for example, it appears that people mind about their employment and housing inequality; in Zimbabwe their actions suggest they mind about land inequality; in Britain, young blacks mind about being stopped by the police as they go about their business; in Sri Lanka, people mind about their employment prospects and their access to higher education. From both a well-being and a political perspective, then, these rather concrete variables may be of more importance than outcome variables, like life expectancy or nutrition levels, or incomes, which are less visible on a day-to-day basis. Moreover, political exclusion and HIs are highly relevant to well-being and to political mobilisation. Political variables include membership of the government, of parliamentary assemblies and local authorities, of the military and of the police. The importance of these variables politically is illustrated by the large part they play in any post-conflict peace negotiations.

Hence, in principle, measurement of HIs needs to extend to a large range of economic, social and political variables, the most relevant ones being context specific. In this spirit, Gurr has pointed to political, social, economic and demographic dimensions of relative deprivation, while Benabou has argued for the need to incorporate political rights in perspectives on inequality, although from an instrumental perspective (Benabou, 2000). Gender inequality analyses almost invariably acknowledge this, exploring employment, education, nutrition, infant survival (UNDP, 1995; Croll, 2000), as well as a set of political variables, e.g. in the UNDP's Gender Empowerment Index (UNDP, 1995).
2. Problems of measuring HIs

In measuring VI, most attention has been devoted to the aggregate measure to be used. This is also an issue for HIs, which we will discuss, but before we get there we must consider the difficult question of how to define the groups.

Given multiple identities and the social construction of identities, there are very few groups where boundaries are clear cut. For example, among Roma people in Eastern Europe, while 90.8% of people interviewed stated that they ‘feel Roma’, only 47.9% reported that they had declared themselves to be Roma in the census (UNDP, 2002). In Guatemala, ‘under certain circumstances an individual can be born indigenous and become Ladino during the course of his or her life’ (Caumartin, 2005: 8). While language is often used as a marker, the 1994 population census indicates that over a quarter of the self-declared indigenous people do not speak an indigenous language. Similar problems arise with most other groups that at first glance seem to be fairly easy to define. For example, Christians in the UK – does this group consist of the Church-going minority, or everyone who doesn’t declare another religion? In Africa, there are many subethnicities and much intermarriage which make boundaries fluid and ill-defined. Moreover, an identity which seems important to the people themselves, or to others, at one point, may become quite trivial - e.g. again in the UK who differentiates between descendants of the Angles and those of the Saxons or the Normans, yet once these were key political distinctions. Moreover, many people declare themselves to have multiple identities (Lee, 2004), so that categorisation of each individual uniquely to one group becomes impossible. With political mobilisation, changes in how people see themselves and others may emerge: the Orang Asli (an active indigenous group in Malaysia), for example, were developed as a group out of at least eighteen different identities to help people mobilise first against the British and subsequently for their rights in independent Malaysia. Muslims in Europe have been mobilised as a group for political reasons as a result of the reactions to the 9/11 bombings and the Iraq war.4

To some extent, then, group boundaries become endogenous to group inequality. If people suffer discrimination (i.e. experience horizontal inequality) they may then feel cultural identity more strongly, particularly if others categorise them into groups for the express purpose of exercising discrimination (thereby creating or enforcing HIs). As Gurr (1993:3) has stated: ‘The psychological bases of group identification are reinforced by cultural, economic and political differentials between the groups and others: treat a group differently by denial or privilege, and its members become more self-conscious about their common bonds and interests. Minimize differences and communal identification becomes less significant as a unifying principle’.

These type of argument could be used to discredit any attempt at measuring HIs. Yet we think this would be wrong: because, even though socially constructed and fluid, these differences do matter to people, as argued above. Moreover, even to test whether they matter or not, and which group distinctions matter, it is necessary to start with some categorisation. But the categorisation should, in so far as is possible, be sensitive to people's self-positioning (and how others’ in society position them). It is also desirable to explore whether adopting different categorisation criteria changes the results. To take the Guatemalan example, although there are many ways of classifying people into groups, the indigenous group, by whatever definition, still comes out at the bottom of the distribution with respect to virtually all variables. In

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4 For example, in Denmark Islamic leaders have ‘urged the country’s 170,000 Muslims to vote against the ruling centre-right coalition’ (Financial Times, 29/30 January 2005).
practice, data deficiencies mean that only rather crude classifications are typically available. But once the importance of the issue is acknowledged, multiple classifications may emerge, as they have, for instance, in the UK census.

Hence, let us assume that we have classified a population into \( n \) groups, and have some information about their performance on \( m \) relevant variables. We then come to the issue of aggregation of HIs which is similar to the issue of identifying a single measure of VI. But there are some important differences. One is that since a group is composed of more than one individual, by definition there are fewer groups than individuals - in fact following the logic of looking only at what seem to be the major felt differences in group membership (and using available data), for most countries there would be rather few groups (sometimes just two or three). The smaller numbers make a big difference, since with so few observations it is possible to look at the individual differences separately and aggregation into a single measure is not only less needed but may actually conceal what is important. In contrast, when we are dealing with a large population of individuals in a society (most country populations exceed half a million and there are a billion or more individuals in China or India), it is essential to find a single (or a few) measures of inequality to be able to comprehend the mass of information available. A second difference between group and individual inequality is that each group is made up of a number of individuals - hence the intra-group distribution may be of interest as well as the inter-group, and it would be possible to include a measure of within group inequality in the measure of each group’s performance. However, this is not desirable because the meaning of the measure would then be no longer clear. Thirdly, the size of each group (i.e. the number of individuals within the group) is likely to differ. Hence one needs to decide whether to look for an aggregate measure which weights according to the size of group. With an unweighted measure, large deviations of very small groups would get the same weight as those of large groups. Yet from a well-being perspective, this represents a different situation in that very different numbers are affected. This would also be true from an instrumental perspective: for example, Posner (2004) has found that the same groups, the Chewas and Tumbukas, have mobilised politically in Malawi, where they form a large proportion of the whole population, but not in Zambia, where their relative size is small. Hence a population weighted index is generally desirable.

A further difference is that much of the discussion of VI assumes the existence of some social welfare function - indeed Atkinson argues that this is essential.\(^5\) Such an SWF is essentially derived from individual preferences/utility functions. There are well known problems in arriving at an SWF for individuals. These would be multiplied in the case of groups, since every group would need to develop a set of preferences (facing the usual problems) and then these would need to be aggregated. Finally, because of the essentially multidimensional nature of HIs, the question arises of whether and how to amalgamate each dimension into a single index. We will put this aside here, and consider how to develop an index for each dimension separately.

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\(^5\) ‘A complete ranking of distributions cannot be reached without fully specifying the form of the social welfare function’ (Atkinson, 1970, quoted in Atkinson, 1983: 30).
Three general principles of a good measure of inequality have been developed for VI, which may be helpful in thinking about a good measure of HI:

1. independence of the distribution from the mean;

2. the principle of transfers (Pigou-Dalton): transfers from a richer person to a poorer person reduce inequality; and

3. the transfer of an equal amount from rich to poor counts for more than one from rich to less rich.

The reason we wish to find a group inequality measure is primarily to explore how far group inequality affects other variables, such as growth, happiness or conflict. Given this objective, in addition to the three principles, which seem reasonable to adopt for group measures too, two further requirements are:

4. in so far as possible, to find a measure which is descriptive, not evaluative. This is not perfectly achievable since any measure involves some implicit valuation, but we aim to minimise this and hence will discard measures which have explicit inequality aversion built in; and

5. to measure group inequality as such, not the contribution of group inequality to either social welfare as a whole (like the gender weighted HDI), nor to income distribution as a whole (as in some important decomposition exercises – see Anand, 1983; Fei, et al, 1978).

Ratios of average performance of pertinent groups are the most straightforward and intuitively appealing measure of group inequality (e.g. ratio of black to white per capita incomes in South Africa). However, such ratios only apply to two groups, and other measures are needed where there are a larger number (as is generally the case). Østby (2003) chooses the two largest groups and calculates the ratios for these groups. In some cases, it seems this choice does not reflect politically salient competition: for example, in Sri Lanka the two largest groups are the low Sinhalese and the upper Sinhalese, while the Tamils are not included, and in South Africa, the two groups are the blacks and coloured, and the whites are not included. Another possibility then would be to choose the two groups that seem to be politically competitive (not necessarily the two largest in population size). However, this would impart a large element of political judgement into the choice. In general, both to assess how fair a society is and to test how far group inequality affects various objectives, there is a need for a synthetic measure which incorporates all group inequalities into a single measure of HI. However, we should bear in mind the possibility that the aggregate synthetic aggregate measure may be influenced by ‘irrelevant’ alternatives. For example, in Malaysia the Chinese/Malay ratios seem to be the politically salient differences. However, a synthetic aggregate measure would also include the Indian population, whose welfare is obviously of importance, but may not be relevant to political stability. While one should not neglect the position of the Indians for this too could become a source of instability, the best way of capturing this might be in the ratio of Indian to Malay performance, rather than a synthetic measure also incorporating the Chinese. Hence for some purposes, especially when the number of relevant groups is small, it may be more helpful to look at simple ratios of each group to the mean, and/or ratios of major groups to each other, as well as the synthetic measures to be discussed below.
Common measures of inequality, mainly devised to measure VI, but that in principle could be used for HI, are:

a. the coefficient of variation, i.e. the variance divided by the mean (COV: Appendix, A1). While this does achieve independence from the mean, it attaches equal weight to redistributions at different income levels, which is a disadvantage according to principle 3 above. Yet, given our objective of arriving at a descriptive, not an evaluative measure, this might not be a strong disadvantage for a group inequality measure. COV involves squaring the deviations from the mean, thus giving more weight to the extremes. Sen has questioned this procedure as being somewhat arbitrary (Sen, 1997: 28). In contrast to some other measures, this only measures differences from the mean, not every difference with every other group. When adapted to measure group inequality (GCOV)\(^6\), this measure might be unweighted (Appendix, B1a), or weighted by the size of the population in each group (Appendix, B1b). Without population weighting, change in the position of a very small group (say accounting for 0.1% of the population) would have the same effect as one involving a large group (accounting for e.g. 60% of the population). These issues will be explored further in section 3.

b. the standard deviation of logarithms. This gives more weight to transfers at the lower end of the distribution – again imparting an evaluative element which we may not want. However, the measure is not sensitive to the unit of measurement, which could be an advantage for HIs where we are incorporating many incommensurate elements. Like the COV, it squares deviations – and because of the use of logs this will further increase the weight given to the extremes. The measure does not always pass the Pigou-Dalton test at high income levels (because the standard deviation is of logs). Like COV, it does not measure each difference against every other individual/group.

c. the Gini coefficient (GINI, Appendix, A2). It has the advantage that it compares every individual (group) with every other and does not square differences. It is especially sensitive to the middle of the distribution. When applied to individuals, it is argued to be not exactly decomposable into within group (WG) and between group (BG) components, unless there is no overlap between groups (Shorrocks, 1994 and Cowell, 1995). However, the Gini can be decomposed into WG and BG components plus a third term, which, according to Yitzhaki and Lerman (1991), can be interpreted as the degree of stratification in society, i.e. groups' isolation from other groups (or the extent of overlapping between the overall rankings of individuals and the relative ranking of the group) in the distribution of the variable/dimension under study. The degree of stratification represented by this third term could be of interest in itself, given its affinity with the concept of polarisation. The aim of the Gini is to measure variance in individual performance, although observations are often grouped by according to achievement on the variable of interest (e.g. all those with 5 years of education might be grouped together when calculating a Gini of human

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\(^6\) In this case one has a choice of variables to use as mean: it might be the mean of the group means, but this would give excessive weight to small groups; alternatively we could use the mean of the whole population. In this case, the measure of inequality would be smaller where there is one very large group, since the performance of this group would tend to dominate the whole and therefore approach the mean. But in a sense this reflects the reality.
capital). For our purpose, we wish to group people by non-economic characteristics (religion, ethnicity, race, etc.) and not by the variable (e.g. years of education) for which the inequality is being calculated. We shall define such a Gini as a group Gini (GGINI, Appendix B2), in contrast to the individual Gini used to measure VI. Such a group Gini will be sensitive to how we draw group boundaries and the number of groups in the population. In general, the greater the number, the more the variance and the higher the group Gini. A big advantage of the Gini applied to VI, is that its long use has made the numbers intuitively comprehensible (i.e. everyone 'knows' that a Gini above 0.6 is very high and one of 0.3 or less is low for developing countries), while this intuitive understanding is absent for the group Gini, though it would gradually develop if the measure were widely adopted.

d. The Theil index (THEIL, Appendix A3). This is especially sensitive to the lower end of the distribution. The Theil is precisely decomposable and it has often been used to divide overall VI inequality into that due to within group inequality (WG) and that due to between group (BG) (Anand, 1983; Heshmati, 2004). However, we are not interested in using the decomposed measure since we are seeking an independent measure of HI, not one that depends on its contribution to total VI. Some researchers are not very fond of the Theil inequality index because of its lack of intuitive appeal (Sen, 1997) \(^7\). However, the Gini coefficient is not completely immune from the same critique, in that it can be argued that the Gini ratio becomes much more appealing when its logic is illustrated through the use of Lorenz curves. In this case, the Gini becomes logically similar to the Theil index, as they both compare population size with share of resources, although the Theil compares each group with the mean (like the GCOV) while the Gini compares with every other group.\(^8\) Thus, the group Theil (GTHEIL, Appendix B3) represents a reasonable alternative to the group Gini in measuring group inequality.

e. Utility based indices, including Dalton's and Atkinson’s index (Dalton (1920); Atkinson (1970). Dalton's measure compares actual aggregate utility with the total level of utility if income were equally divided, assuming diminishing marginal utility. Atkinson criticised it for changing according to linear transformations of the utility function and developed his own equally distributed equivalent measure, which is not subject to this criticism. Both are explicitly normative measures. Atkinson’s equally distributed equivalent measure varies according to the value assumed for inequality aversion, to be derived from a SWF. As noted above, there are severe problems in arriving at an SWF based on group preferences. Moreover, the normative basis of these measures makes them inappropriate for a group inequality measure intended to be primarily descriptive.

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\(^7\) ‘The average of the logarithms of the reciprocals of income shares weighted by income shares is not exactly overflowing with intuitive sense’ (Sen, 1997: 36).

\(^8\) In fact, Theil can be interpreted as a weighted summation of the (log) ratio between each groups' share of a certain asset to the group’s population share, with weights equal to the group’s asset share (Conceicao and Ferreira, 2000).
While the measures above were developed for VI but could be used to measure HI, other measures have been specifically designed to explore aspects of group inequality.

f. The Estaban-Ray (ER) polarisation index (Esteban and Ray, 1994; Duclos et al, 2004; ER(1.5), Appendix B4). The ER index is similar to a group Gini\(^9\), weighted by population, but includes an index – 1+\(\alpha\), where 1 ≤ \(\alpha\) <1.6 – which is higher the greater the weight attached to the share of group population in total population. A common value for \(\alpha\) is 1.5 (Kanbur & Zhang, 1999). This index is not decomposable. The point of \(\alpha\) is to increase the weight given to large groups, so that the index rises as the population is distributed among fewer and more equally sized groups. Consequently, two populations might have the same value of the index, despite one having less variance in resource access or incomes between the groups than the other, so long as the one with less variance had a smaller number of larger groups than the other. The idea behind this measure is that demographic polarisation is likely to increase the likelihood of conflict. Estaban and Ray in fact argue that the index is not a measure of group inequality but of societal polarisation. The disadvantage of the measure from our perspective is first that it includes two elements that we wish to explore separately – the size distribution of the groups and the extent of inequality among them; and secondly, that there is an arbitrariness (and an evaluative element) about the choice of \(\alpha\). The ER measure also violates the Pigou-Dalton condition because increased demographic polarisation can offset a given income transfer from a richer to a poorer group.

g. The Zhang/Kanbur (ZK) polarisation index is the ratio of \(BG/WG\), using Theil to measure \(BG\) and \(WG\). This is higher the more HIs contribute to overall inequality. The problem with this, from our perspective, is that the size of the measure will vary according to within group variance. Thus the same between group variance will lead to different measures according to WG. We wish to separate our measure of HIs from what is happening within the group. Within group variance is of interest – a key question is whether between group inequality matters less for group conflict when there is much within group inequality, but we need to measure the two types of inequality independently to be able to investigate this question.

h. The odds ratio suggested by Chakrabarty (2001). This measure calculates the odds of individuals in a particular group falling into a particular category (e.g. rich or poor), and then expresses the group differences as the ratio of these odds. This basically resembles the method of using simple ratios of performance, except that the performance of each group is not the average but the odds of being poor, calculated as a proportion of the poor to the total population in each group. Hence it requires somewhat more data than the ratios of average proportion. Since it is designed to measure differences between two groups, some other method would need to be introduced to generalise to many groups (e.g. by adopting a Gini of the odds).

Work in this area has tended to be less interested in measuring group inequality as such, and more interested in devising a measure of general welfare that allows for group inequality – for example, Anand and Sen (1995) in their gender weighted HDI

\(^9\) Where \(\alpha = 0\), this is equivalent to the group Gini.
(the Gender Development Index or GDI). This is a social welfare evaluation, aiming to weight trade-offs between higher average achievement and more inequality between genders. Each component of HDI is modified to take into account gender inequalities, by estimating an ‘equally distributed equivalent achievement’. Like Atkinson’s equally distributed equivalent measure, it contains a choice of weights, $\varepsilon$, to represent inequality aversion. When $\varepsilon = 0$, there is no weighting and as $\varepsilon$ tends to infinity only the lowest value is included. This is a good method for producing a group weighted welfare function, but not for a descriptive measure of inequality.

Similarly, Majumdar & Subramanian (2001) explore group inequality because it is ‘politically more salient and consequential than interpersonal comparison’. They adjust a measure of deprivation, or capability failure, by a group deprivation index. The capability index is a weighted index of several deprivations. The index is adjusted by a formula for the deprivation of the particular group. Again the aim is to adjust aggregate welfare rather than to describe group inequality. They do not include an inequality aversion index, although one is implicit.

**Conclusions on approaches to measuring group inequality:**

The first consideration is that we aim - as far as possible - to get a descriptive measure of HIs – hence we reject those measures which involve a strong explicitly evaluative element - i.e. the Dalton/Atkinson measures. This is also a problem with the ER polarisation index. The standard deviation of logarithms gives a lot of weight to extreme observations. Of course, the other measures contain some element of valuation – e.g. the GCOV by squaring the observations gives more weight to observations further from the mean and the Gini gives relatively more weight to the middle of the distribution.

Secondly, we are keen to have a measure of HI which is distinct from other influences such as VI and population distribution. This is a problem with both the ER and the ZK index. The ER index combines two elements: inequality and population polarisation. Hence the same distance between groups would get greater weight the more the population is demographically polarised. The ZK index incorporates both BG and WG inequality into the index. Thus the same HIs would get different values according to the extent of WG. From our perspective this is not desirable, as in empirical work we wish to describe the extent of between group inequality, and to test what impact this has on various outcomes (e.g. growth; conflict; happiness). While the impact of HIs may vary according to the extent of heterogeneity within the group (WG) or the extent of demographic polarisation, we wish to test both these elements separately, which is not possible if they are incorporated in a single index. BG, by itself, as defined by Zhang and Kanbur is not a pure measure of HIs, as it represents the contribution of HIs to total VI in a society, so that the value of BG would go down as WG goes up, for the same HI. Thirdly, as noted, we need a measure which captures inequality among more than two groups. Hence the odds ratio by itself is insufficient, though it could provide the inputs into another measure, such as a group Gini, which aggregates across a number of groups.

This leaves us with three measures out of those discussed above:

1. The coefficient of variation among groups (GCOV), which can be weighted according to the population of each group. Population weighting seems to be desirable because otherwise small groups can have a disproportionate impact on the measure of HI. Moreover, while all measures of HI are sensitive to the way group boundaries are drawn, this is arguably less so where groups are weighted by population size. The GCOV is a straightforward measure,
basically descriptive (although, of course, some implicit evaluation is involved in any measure). The coefficient of variation is a common measure of regional disparities, e.g. Quah (1996) uses the standard deviation.

2. The Group Gini (GGINI) coefficient, where groups are defined according to some relevant "non-economic" dimensions such as ethnicity, race, gender or region, not, as is more usual, in terms of the same dimension/metric used to summarize the distance between them. 10

3. The Group Theil (GTHEIL), again where groups are defined by social identity of some sort.

3. Applications: South Africa, the United States and Indonesia

In this section of the paper we explore whether selected measures of VI and HI in practice move together or differently on the basis of data from South Africa, the US and Indonesia. This gives insight into (a) how far it matters which measure we use; and (b) which measure seems to fit our intuitive understanding of the situation.

Longitudinal income data from South Africa (Figure 1) shows that measures used for HI over time can produce divergent trends: in particular an unweighted GCOV among racial groups shows a rise in HI since 1975, while the population weighted GCOV, the GGINI and the ER(1.5) measure all show a trend fall. The fastest fall was for ER(1.5) because of the inequality aversion built into that measure. The difference between the unweighted and weighted measures is accounted for by the fact that while Black African incomes rose at an above average rate and got nearer to the mean, white incomes also rose faster than the mean. This was possible because the growing proportion of the total population accounted for by Black Africans pulled the average growth of incomes below that of either of the two groups. The increased deviation of white incomes from the mean was more than enough to offset the decreased deviation of Black African incomes, especially since White incomes were way above the mean and differences from the mean are squared with the GCOV. The basic problem is that the mean is calculated on the basis of population weighting so it cannot appropriately be combined with an unweighted measure. The increase in the unweighted GCOV is counterintuitive: the ratio of black white income per capita fell significantly.

Similarly, we find conflicting trends for the US from 1967-2001 for black/white/other inequality (Figure 2) The population unweighted GCOV shows fluctuations but little change. However, there appears to be a slightly rising trend for the population weighted GCOV. The group Gini shows a bigger rise, and the sharpest rise is for ER(1.5), which again magnifies the increase in inequality. In this case, the basic reason for the divergence is that a rise in the black population’s income relative to the mean was just about offset by a movement in the rest of the population’s incomes away from the mean in the case of the unweighted GCOV, but the small proportionate size of the black population – 8.3% in 2001 – meant that group distribution worsened when measures are population weighted.

10 Using the same variable to define the group and to measure variance is fairly common - for example, grouping by education levels to construct Gini coefficients for human capital (Thomas et al., 2000).
Figure 1: Measures of Horizontal Inequality in South Africa, 1970 –2000

Source: Calculated from data in Louw & van den Berg (2004).11

Figure 2: Measures of Horizontal Inequality in the United States, 1967-2001

Source: Calculated from US Census Bureau data

11 Louw and van den Berg give two separates estimates for 2000 – one ‘optimistic’, the other ‘pessimistic’. For the purposes of this graph, the mid-way point between the two estimates was taken.
The divergences in the measures in the two sets of data – and the reasons for them – lead us to conclude that the population unweighted GCOV is generally not a satisfactory measure. The two diagrams also show the movement of the ratio of white/black incomes in South Africa and the US. In both cases, this shows a greater movement towards equality than any of the more complex measures, which also include other groups. It could be argued that this ratio which is of greatest interest, being the most politically provocative and possibly most related to the personal wellbeing of the deprived group. If this is the case, the measures we have been discussing may actually obfuscate rather than clarify the situation.

We explore these relationships further using cross-sectional data from Indonesian censal data in 1990 and 1995 at the district level, with 282 and 304 observation respectively. In the correlation matrix shown in Table 1 we have calculated measures of VI and HI based on years of education for 88 religiously diverse districts in Indonesia. The diagonal data in bold shows the correlation of each indicator with its own value in the two years. Table 2 does the same as Table 1 but includes a further 186 homogeneous districts. The following interesting results emerge:

1. For both cases, the indices of vertical inequality – Gini, Theil and the coefficient of variation (COV) – are nearly perfectly correlated in both years, indicating that for this data set it is not important which measure is selected;

2. In both tables, HI measures are also highly correlated with each other in both years, with coefficients ranging between 0.76 and 0.96 – again suggesting that the choice of group inequality indicator may not be important;

3. The correlations between measures of VI and HI are much smaller than the correlations within either the VI or HI measures. In Table 1, there is generally a positive correlation between the VI and HI measures. The ZK measure is the only one to show weak correlation – the way it has been constructed accounts for this since it eliminates the element of VI accounted for by HI. For the other measures of group inequality, the correlation between HI and VI ranges from 0.28 to 0.54. In these estimates, however, we excluded ethnically/religiously homogeneous districts. When these are included (Table 2) the correlation between each measure of HI and VI is negative, much smaller in magnitude and generally statistically non-significant at standard levels especially when 1990 data are used. It is relevant to include homogeneous districts if one wants to explore how far a measure of VI can proxy for HI in society as a whole. However, if the purpose is to explain intercommunal conflict, including homogeneous districts will spuriously increase the significance of HI because both communal conflict and HI will be zero in such districts. A very low correlation between VI and HI, (r= 0.03), was also found by Østby (2003) in a cross-country analysis;

12 The 1995 dataset contains 304 rather than 282 districts because the province of East Timor was excluded from the 1990 dataset due to under-sampling.
13 Groups are defined by religion, which in both samples is categorized as: Islam, Catholic, Protestant/Other Christian, Hindu, Buddhist and Other.
Table 1: Correlation matrix between inequality measures based on years of education in Indonesia: N=88 districts

<table>
<thead>
<tr>
<th>YEAR</th>
<th>VERTICAL INEQUALITY</th>
<th>HORIZONTAL INEQUALITY</th>
<th>FRAG.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GINI</td>
<td>THEIL</td>
<td>COV</td>
</tr>
<tr>
<td>GINI</td>
<td>0.87 ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>THEIL</td>
<td></td>
<td>0.99 ***</td>
<td>0.83 ***</td>
</tr>
<tr>
<td></td>
<td>0.98 ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>COV</td>
<td>1995</td>
<td>0.99 ***</td>
<td>0.99 ***</td>
</tr>
<tr>
<td></td>
<td>1990</td>
<td>0.99 ***</td>
<td>0.99 ***</td>
</tr>
<tr>
<td>GGINI</td>
<td>1995</td>
<td>0.40 ***</td>
<td>0.42 ***</td>
</tr>
<tr>
<td></td>
<td>1990</td>
<td>0.41 ***</td>
<td>0.36 ***</td>
</tr>
<tr>
<td>GTHEIL</td>
<td>1995</td>
<td>0.52 ***</td>
<td>0.54 ***</td>
</tr>
<tr>
<td></td>
<td>1990</td>
<td>0.43 ***</td>
<td>0.39 ***</td>
</tr>
<tr>
<td>ER1.5</td>
<td>1995</td>
<td>0.29 ***</td>
<td>0.30 ***</td>
</tr>
<tr>
<td></td>
<td>1990</td>
<td>0.33 ***</td>
<td>0.29 ***</td>
</tr>
<tr>
<td>GCOV</td>
<td>1995</td>
<td>0.46 ***</td>
<td>0.47 ***</td>
</tr>
<tr>
<td></td>
<td>1990</td>
<td>0.47 ***</td>
<td>0.42 ***</td>
</tr>
<tr>
<td>ZK</td>
<td>1995</td>
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<td>0.17</td>
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<tr>
<td></td>
<td>1990</td>
<td>0.21 **</td>
<td>0.18 *</td>
</tr>
<tr>
<td>ERF</td>
<td>1995</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>1990</td>
<td>0.15</td>
<td>0.13</td>
</tr>
</tbody>
</table>

¹ Only religiously diverse districts in both years are considered (ERF>=0.1). ***, **, * indicate significance at 1%, 5%, and 10% level, respectively
² The diagonal elements shown in bold for each index are the correlation coefficients between 1990 and 1995
Source: Census 1990, Supas 1995
Table 2: Correlation matrix between inequality measures based on years of education in Indonesia: N=274 districts ¹ ²

<table>
<thead>
<tr>
<th>YEAR</th>
<th>VERTICAL INEQUALITY</th>
<th>HORIZONTAL INEQUALITY</th>
<th>FRAG.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GINI</td>
<td>THEIL</td>
<td>COV</td>
</tr>
<tr>
<td>GINI</td>
<td>0.90 ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>THEIL</td>
<td></td>
<td>0.91 ***</td>
<td></td>
</tr>
<tr>
<td>COV</td>
<td>0.99 ***</td>
<td>0.99 ***</td>
<td>0.91 ***</td>
</tr>
<tr>
<td>GGINI</td>
<td>-0.05</td>
<td>-0.06</td>
<td>-0.07</td>
</tr>
<tr>
<td>GGINI</td>
<td>-0.03</td>
<td>-0.07</td>
<td>-0.05</td>
</tr>
<tr>
<td>GTHEIL</td>
<td>0.13 **</td>
<td>0.11 *</td>
<td>0.11 *</td>
</tr>
<tr>
<td>ER1.5</td>
<td>-0.10 *</td>
<td>-0.12 *</td>
<td>-0.13 **</td>
</tr>
<tr>
<td>GCOV</td>
<td>-0.03</td>
<td>-0.04</td>
<td>-0.05</td>
</tr>
<tr>
<td>ZK</td>
<td>-0.12 **</td>
<td>-0.13 **</td>
<td>-0.14 **</td>
</tr>
<tr>
<td>ERF</td>
<td>-0.24 ***</td>
<td>-0.24 ***</td>
<td>-0.26 ***</td>
</tr>
</tbody>
</table>

¹ The sample includes the religiously homogenous districts. ***, **, * indicate statistical significance at 1%, 5%, and 10% level, respectively
² The diagonal elements shown in bold for each index are the correlation coefficients between 1990 and 1995
Source: Census 1990, Supas 1995
### Table 3: Correlation matrix between inequality measures based on income in Indonesia: N=154 districts

<table>
<thead>
<tr>
<th>VERTICAL INEQUALITY</th>
<th>HORIZONTAL INEQUALITY</th>
<th>FRAG.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GINI</strong></td>
<td><strong>THEIL</strong></td>
<td><strong>COV</strong></td>
</tr>
<tr>
<td>1.00</td>
<td>**0.94 *****</td>
<td><strong>1.00</strong></td>
</tr>
<tr>
<td>**0.94 *****</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>**0.81 *****</td>
<td>**0.94 *****</td>
<td><strong>1.00</strong></td>
</tr>
</tbody>
</table>

| **GGINI**           | **GTHEIL**            | **GCOV** |
| 0.25 ***            | 0.34 ***              | 0.40 *** |
| **0.36 *****        | 0.49 ***              | 0.56 *** |
| **0.37 *****        | 0.46 ***              | 0.52 *** |

| **ER1.5**           | **ZK**                |
| 0.32 ***            | 0.17 **               |
| **0.01**            | **0.10**              |

| **ZK**              | **ERF**               |
| 0.01                | **0.17 **             |
| **0.01**            | **-0.09**             |

| **ERF**             |                     |
| -0.19 **            | -0.19 **             |
| -0.17 **            | **0.17 **            |

**1** Only religiously diverse districts are considered (ERF>=0.1).

***, **, * indicate significance at 1%, 5%, and 10% level, respectively

Source: SUPAS 1995

### Table 4: Correlation matrix between inequality measures based on income in Indonesia: N=304 districts

<table>
<thead>
<tr>
<th>VERTICAL INEQUALITY</th>
<th>HORIZONTAL INEQUALITY</th>
<th>FRAG.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GINI</strong></td>
<td><strong>THEIL</strong></td>
<td><strong>COV</strong></td>
</tr>
<tr>
<td>1.00</td>
<td>**0.85 *****</td>
<td><strong>1.00</strong></td>
</tr>
<tr>
<td>**0.75 *****</td>
<td>0.94 ***</td>
<td><strong>1.00</strong></td>
</tr>
</tbody>
</table>

| **GGINI**           | **GTHEIL**            | **GCOV** |
| 0.07                | 0.10 *                | 0.16 *** |
| **0.25 *****        | 0.25 ***              | 0.30 *** |
| **0.22 *****        | 0.20 ***              | 0.25 *** |

| **ER1.5**           | **ZK**                |
| 0.13 **            | 0.15 ***              |
| **0.00**           | 0.04                  |

| **ZK**              | **ERF**               |
| 0.00                | **0.04**              |
| **0.10 **           | **0.81**              |

| **ERF**             |                     |
| -0.24 **            | -0.15 **             |
| -0.12 **            | **0.55 **            |

**1** All districts are considered.

***, **, * indicate significance at 1%, 5%, and 10% level, respectively

Source: SUPAS 1995
4. HI measures are positively correlated with ethno-religious (demographic) fragmentation (ERF, Appendix C1). The GGINI measure appear to be significantly more sensitive to fragmentation than other HI indicators. This means that group inequalities increase as the number of groups in a society increase, which, perhaps, is intuitively plausible; and

5. HI measures tend to be less correlated over time than VI indicators, although all show a quite high correlation over time. The lowest correlation over time was with ER(1.5), with a correlation coefficient of 0.55 when religiously homogenous districts are excluded – probably because of population movements over this time. The difference in correlations over time between VI and HI measures implies that HI and VI dynamics differed across Indonesian districts between 1990 and 1995, with HI changing more than VI inequality in this case. From one perspective this is surprising – if group inequality stemmed from discrimination or asset inequality one would not expect this to change quickly while individuals might move more readily up or down the income scale. However, the very large numbers involved in VI mean that upward and downward changes are more likely to offset each other than in the case of HI which is based on small numbers of groups. The data on South African white/black income ratios shown below (Figure 1) similarly shows much more change than VI in South Africa over the same period.

Carrying out a similar exercise with income data for 304 districts in Indonesia in 1995 (Tables 3 and 4) broadly confirms the results of Tables 1 and 2. Again, there are quite high correlations among alternative measures of HI, but much lower correlations between HI and VI.

4. Some conclusions

This paper has argued that group inequality is important because it can affect happiness, efficiency, and political stability. The paper explores alternative approaches to measurement of group inequality. We acknowledge the severe problem of defining group boundaries, since identities are fluid, multiple and may even be endogenous. Nonetheless felt differences seem important enough and clear enough in many societies to make it possible to measure group performance so long as one is sensitive to the possibility and implications of alterations in group boundaries.

We reviewed alternative measures of group inequality and compared them using some Indonesian data for two dates, as well as some longitudinal data for South Africa and the US. The Indonesian data show that each of the measures of vertical inequality is highly correlated with every other one; the measures of group inequality are also correlated with each other but less strongly. When homogenous districts are excluded, there is a positive correlation between horizontal and vertical inequality for an education variable (not exceeding 0.54), but when homogeneous districts are included this correlation becomes very low. Both VI and HI show persistence over time. But for education, within district HI changes more over time than VI, which could be important politically as durable HI is likely to be particularly provocative. We have not been able to investigate this issue using income.

Data on South Africa and the US, shows that different HI measures can generate different trends. One important difference is that the population unweighted GCOV moves in a different direction from the population weighted measure, the group Gini or the ER(1.5). In the case of South Africa, the unweighted GCOV shows increasing HI, while the other measures show some decline. In the case of the US it is the other
way round, with the unweighted GCOV showing little change and the other measures showing some increase. In the South African case, this arises from an oddity in the population unweighted index – that it sums the unweighted squares of deviation from the mean, where the mean itself is population weighted; in the US case, it arises because of the relatively small size of the black population. The population weighted GCOV, the group Gini and the ER(1.5) measures however, generally move in the same direction, with the GCOV showing the least change, the ER(1.5) the most and the group Gini being in the middle. In principle, weighted GCOV could move in a different direction from the group Gini and ER(1.5), while generally one would expect the latter two measures to move in the same direction. If one is particularly concerned with the position of one group, or of that group relative to one other group, the clearest way to present the data is to take the ratio of the performance of the group to the mean, or the ratio of the performance of the two groups, rather than some group measure of inequality which also introduces other groups and may conceal the variable of interest. This was indicated by both US and South African data.

The value of a measure of inequality depends on the purpose for which one wants it. One purpose is to make a general statement about whether a society is getting fairer or less fair over time from a group perspective. Another objective we have is to identify a variable which will enable us to test whether particular inequalities are correlated with other events, such as conflict, criminality, and unhappiness. For both purposes, group weighting by population would seem desirable. The first objective requires a measure which is widely understood – which could be a problem for all the suggested measures. From the perspective of assessing how fair a society is, the measure of inequality could include some element of evaluation (as in ER where \( \alpha > 0 \)). However, unless this valuation is widely understood and shared, it may be better to present the data without such a valuation element.

For the second objective – to identify a variable we can use to explore consequences of HIs – it is preferable to exclude elements of evaluation, as far as possible, and to have a separate measure of HI and VI. The ZK measure does not present separate measures of between group and within group inequality, and is consequently ruled out from this perspective. Hence, the population weighted GCOV or the Group Gini appear to be the preferred measures for this objective.

**References**


Appendix

A. Vertical inequality

A1 \[ \mathrm{COV} = \frac{1}{\bar{y}} \left( \frac{1}{n-1} \sum_i^n (y_i - \bar{y})^2 \right)^{\frac{1}{2}} \]

A2 \[ \mathrm{GINI} = \frac{1}{2n^2 \bar{y}} \sum_i^n \sum_j^n |y_i - y_j| \]

A3 \[ \mathrm{THEIL} = \frac{1}{n} \sum_i^n \frac{y_i}{\bar{y}} \log \left( \frac{y_i}{\bar{y}} \right) \]

where \( \bar{y} \) is the sample mean, \( y_i \) is the number of years of education of individual \( i \) and \( n \) is the sample size.

B. Horizontal inequality

B1a Unweighted GCOV = \[ \frac{1}{\bar{y}} \left( \frac{1}{R-1} \sum_r^R (\bar{y}_r - \bar{y})^2 \right)^{\frac{1}{2}} \]

B1b Weighted GCOV = \[ \frac{1}{\bar{y}} \left( \sum_r^R p_r (\bar{y}_r - \bar{y})^2 \right)^{\frac{1}{2}} \]

B2 \[ \mathrm{GGINI} = \frac{1}{\bar{y}} \sum_r^R \sum_s^S p_r p_s |\bar{y}_r - \bar{y}_s| \]

B3 \[ \mathrm{GTHEIL} = \sum_r^R p_r \frac{\bar{y}_r}{\bar{y}} \log \left( \frac{\bar{y}_r}{\bar{y}} \right) \]

B4 \[ \mathrm{ER} (k, \alpha) = k \sum_r^R \sum_s^S p_r \frac{1}{\bar{y}} \log \left( \frac{y_s}{\bar{y}} \right), \quad k = \frac{1}{\bar{y}}, \ \alpha = 1.5 \]

B5 \[ \mathrm{ZK} = \frac{\mathrm{BG}}{\mathrm{WG}}, \quad \mathrm{WG} = \sum_r^R \frac{\bar{y}_r}{Y} \left[ \frac{1}{n_r} \sum_i^n \frac{y_{ir}}{\bar{y}_r} \log \left( \frac{y_{ir}}{\bar{y}_r} \right) \right], \ \mathrm{BG} = \mathrm{THEIL} - \mathrm{WG} \]

where \( \bar{y}_r = \frac{1}{n_r} \sum_i^n y_{ir} \) is group \( r \) mean value, \( p_r \) is group \( r \) population share, \( y_{ir} \) is the number of years of education of the \( i^{th} \) member of group \( r \), \( Y_r \) is the total number of years of education of group \( r \), and \( Y \) is the total number of years of education in the sample.
C. Demographic fragmentation

C1 \[ \text{ERF} = 1 - \sum_{j} p_{j}^{2} \]